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THE ODD GENERALIZED EXPONENTIAL LOG-LOGISTIC DISTRIBUTION GROUP ACCEPTANCE SAMPLING PLAN

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ABSTRACT

In this manuscript, a group acceptance sampling plan (GASP) is developed when the lifetime of the items follows odd generalized exponential log-logistic distribution (OGELLD), the multiple number of items as a group can be tested simultaneously in a tester. The design parameters such as the minimum group size and the acceptance number are derived when the consumer's risk and the test termination time are specified. The operating characteristic (OC) function values are calculated (intended) according to various quality levels and the minimum ratios of the true average life to the specified average life at the specified producer's risk are derived. The methodology is illustrated through real data.

Key words: odd generalized exponential log-logistic distribution, group acceptance sampling plan, truncated life test.

1. Introduction

In the present highly competitive global market, different items/products are categorized on several factors of end products. One such factor is quality/durability of a product, which can be examined through most of statistical quality control techniques, which are the two important statistical tools for ensuring the quality of the product, which are (i) Process control and (ii) Product control. In acceptance sampling plans for a truncated life test, the utmost issue is to determine the sample size from a lot under cogitation. In most of the statistical quality control experiments, it is not possible to perform 100% inspection due to various reasons. It is implicitly assumed in the usual sampling plan; the decision of accepting or rejecting a lot is on the basis of a sample of items. To save cost and time in the life test, it is very often to put a number of items in a tester. In this life test, a tester is called a group and the number of items in each tester is called the group size. The acceptance sampling via the group life test is called the group acceptance sampling plan (GASP), which is also often enacted under a truncated life test. For such a type of test, the determination of the sample size is equivalent

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to fixation of the number of groups. This type of testers is customarily used in the case of the so-called sudden death testing, which is discussed by Pascual and Meeker (1998) and Vlcek *et al*. (2004). Jun *et al.* (2006) introduced this group concept into acceptance sampling plan and developed variable sampling plans for sudden death testing for the Weibull distribution. GASPs under a truncated life test have been studied by many researchers for different lifetime distributions.

A group sampling plan based on truncated life test based on the gamma distributed items was contemplated by Aslam *et al.* (2009). Aslam and Jun (2009a, 2009b) developed a group acceptance sampling plan for truncated life tests based on the inverse Rayleigh, log-logistic and Weibull distributions. Balamurali and Jun (2009) proposed a repetitive group sampling procedure for variables inspection. Rao (2009, 2010) presented a group acceptance sampling plans for lifetimes following a generalized exponential distribution and Marshall-Olkin extended Lomax distribution. Group acceptance sampling plans for Pareto distribution of the second kind was discussed by Aslam *et al.* (2010). Radhakrishnan and Alagirisamy (2011) developed a group acceptance sampling plan using weighted binomial distribution. Ramaswamy and Anburajan (2012) developed a group acceptance sampling plan using weighted binomial on truncated life tests for inverse Rayleigh and log-logistic distributions. A group acceptance sampling plan using weighted binomial for a truncated life test when the lifetime of an item follows exponential and Weibull distributions was contemplated by Anburajan and Ramaswamy (2015). Rao and Ramesh (2015) considered a group acceptance sampling plans for exponentiated half logistic distribution. Rao *et al.* (2016) studied group acceptance sampling plans for lifetimes following an exponentiated Fréchet distribution. Rao and Rao (2016) developed a two-stage group acceptance sampling plan based on life tests for half logistic distribution. Group acceptance sampling plans for odds exponential log-logistic distribution and Type-II generalized log-logistic distribution were contemplated by Rosaiah *et al.* (2016a, 2016b).

To reduce the experimental time and cost, GASPs have been used (see Jun *et al.,* (2006)). In this case, the total number of items n to be tested is divided into equal-sized groups according to the number of available experimental testers. Suppose 'r' items are in each group and there are a total of 'g' groups, then $n = rg$. The items in each group are tested independently under identical environmental conditions. Moreover, all the testers run simultaneously. The experiment is stopped at a pre-specified time t. If 'c' is the acceptance number for this experiment, then a lot is accepted if the recorded number of failures in each group is less than 'c' during the experimental time t. The single sampling plans handle this problem by assuming a parametric model for the lifetime distribution and then deriving the minimum sample size 'n' needed to ensure certain mean or median life of the items under investigation. It is further assumed that the experimental time and the number of items in each group are prefixed in advance. Since n=rg, determining 'n' is equivalent to determining 'g'.

The main purpose of this manuscript is to develop the GASPs for the odd generalized exponential log-logistic distribution (OGELLD). Gupta (1962) suggested that for a skewed distribution, the median represents a better quality parameter than the mean. On the other hand, for symmetric distribution, mean is preferable to use as a quality parameter. Since OGELLD is a skewed distribution, we prefer to use the percentile point as the quality parameter and it will be denoted by t_q.

The rest of this manuscript is organized as follows. In Section 2, we describe concisely the OGELLD distribution. The design of group acceptance sampling for lifetime percentiles under a truncated life test is discussed in Section 3. In Section 4, description of the proposed methodology with real data example is presented. A comparison of distributions is discussed in Section 5. Finally, conclusions are made in Section 6.

2. The Odd Generalized Exponential Log-Logistic Distribution (OGELLD)

In this section, we provide a brief summary of the odd generalized exponential log-logistic distribution (OGELLD). The OGELLD was introduced and studied quite extensively by Rosaiah *et al.* (2016c). The probability density function (pdf) and cumulative distribution function (cdf) of OGELLD respectively are given as follows:

$$f(\mathrm{t};\sigma,\lambda,\theta,\gamma)=\frac{\gamma\theta}{\lambda\sigma}\left(t/\sigma\right)^{\theta-1}\left[1-e^{\frac{-1}{\lambda}\left(t/\sigma\right)^{\theta}}\right]^{\gamma-1}e^{\frac{-1}{\lambda}\left(t/\sigma\right)^{\theta}},\ t>0,\sigma,\lambda,\theta,\gamma>0 \tag{1}$$

and $$F(\mathrm{t};\sigma,\lambda,\theta,\gamma)=\left[1-e^{\frac{-1}{\lambda}\left(t/\sigma\right)^{\theta}}\right]^{\gamma},t>0,\ \sigma,\lambda,\theta,\gamma>0. \tag{2}$$

where σ, λ are the scale parameters and θ,γ are the shape parameters. The 100*q*-th percentile of the OGELLD is given as:

$$t_q=\sigma\eta_q,\quad \text{where}\quad \eta_q=\left[-\lambda\ln(1-q^{1/\gamma})\right]^{1/\theta}. \tag{3}$$

Hence, for the fixed values of $\lambda=\lambda_0$ and $\theta=\theta_0$, the quantile t_q given in Equation (3) is the function of scale parameter $\sigma=\sigma_0$, that is $t_q\geq t_q^0\Leftrightarrow\sigma\geq\sigma_0$,

where $$\sigma_0=\frac{t_q^0}{\left[-\lambda_0\ln(1-q^{1/\gamma_0})\right]^{1/\theta_0}}. \tag{4}$$

Note that σ_0 also depends on λ_0 and θ_0, to build up acceptance sampling plans for the OGELLD ascertain $t_q\geq t_q^0$, equivalently that σ exceeds σ_0.

3. The Group Acceptance Sampling Plan (GASP)

In this section, we provide group acceptance sampling plans (GASPs) when a lifetime of the product is an OGELLD with known scale and shape parameters λ, θ. We propose the GASP under the truncated life test, which is based on the total number of failures from all groups. The procedure of the proposed plan is as follows [Aslam *et al*. (2011a)]:

- Step 1: Randomly draw a sample of size *n* from a production lot, allocate *r* items to each of *g* groups (or testers) so that *n* = *rg* and put them on test until the pre-determined t_0 units of time.
- Step 2: Accept the lot when the number of failures from all *g* groups is smaller than or equal to *c*. Truncate the test and reject the lot as soon as the number of failures from all *g* groups is larger than *c* before t_0.

The probability of accepting a lot for the group sampling plan based on the number of failures from all groups under a truncated life test at the test time schedule t_0 is

$$\mathrm{P}_a(p) = \sum_{i=0}^{c} \binom{rg}{i} p^i \left(1-p\right)^{rg-i} \tag{5}$$

where *'g'* is the number of groups, *'c'* is the acceptance number, *'r'* is the group size, and *'p'* is the probability of getting a failure within the life test schedule, t_0. Since the product lifetime follows OGELLD, we have $p = F\left(t_0\right)$. Usually, it would be convenient to determine the experiment termination time, t_0, as $t_0 = \delta_q^0 t_q^0$ for a constant δ_q^0 and the targeted 100q-th lifetime percentile, t_q^0. Let t_q be the true 100q-th lifetime percentile. Then, *p* can be rewritten as

$$p = \left[1 - e^{-\frac{1}{\lambda}\left(\frac{t_0}{\sigma}\right)^{\theta}}\right]^{\gamma} = \left[1 - e^{(-1/\lambda)\left(\eta_q \delta_q^0 / \left(t_q / t_q^0\right)\right)^{\theta}}\right]^{\gamma} \tag{6}$$

In order to find the design parameters of the proposed GASP, we prefer the approach based on two points on the OC curve by considering the producer's and consumer's risks. In our approach, the quality level is measured through the ratio of its percentile lifetime to the lifetime, t_q / t_q^0. These percentile ratios are very helpful to the producer to enhance the quality of products. From the producer's perspective, the probability of lot acceptance should be at least $1-\alpha$ at the acceptable reliability level (ARL), p_1. Thus, the producer demands that a lot should be accepted at various levels, say t_q / t_q^0 =2, 4, 6, 8, 10 in Equation (6). On the other hand, from the consumer's viewpoint, the lot rejection probability should be at most β at the lot tolerance reliability level (LTRL), p_2. In this way, the consumer considers that a lot should be rejected when t_q / t_q^0 =1, in Equation (6).

$$\sum_{i=0}^{c} \binom{rg}{i} p_1^i (1-p_1)^{rg-i} \geq 1-\alpha \tag{7}$$

$$\sum_{i=0}^{c} \binom{rg}{i} p_2^i (1-p_2)^{rg-i} \leq \beta \tag{8}$$

where p_1 and p_2 are given by

$$p_1 = \left[1-e^{(-1/\lambda)\left(\eta_q \delta_q^0 / \left(t_q / t_q^0\right)\right)^{\theta}}\right]^{\gamma} \text{ and } p_2 = \left[1-e^{(-1/\lambda)\left(\eta_q \delta_q^0\right)^{\theta}}\right]^{\gamma} \tag{9}$$

The plan parametric quantities for distinct values of parameters $\lambda, \theta \text{ and } \gamma$ are constructed. Given the producer's risk $\alpha = 0.05$ and termination time schedule $t_0 = \delta_q t_q^0$ with $\delta_q^0 = 0.5 \text{ or } 1,$ the three parameters of the proposed group acceptance sampling plan under the truncated life test at the pre-specified time, t_0, with $\lambda = \theta = \gamma = 2$ are obtained according to the consumer's risk $\beta = 0.25, 0.10, 0.05 \text{ and } 0.01$ for 50th and 25th percentiles, which are shown in Tables 1 to 4.

4. Description of the proposed methodology with real data example

We demonstrate the application of the proposed group acceptance sampling plan for the OGELLD using real lifetime data set from Dey *et al*. (2018), which represent the survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli. Guinea pigs are known to have high susceptibility to human tuberculosis, which is one of the reasons to choose guinea pigs for such a study. Here, we consider only the study where all animals in a single cage are under the same regimen. The data were observed and reported by Bjerkedal (1960). For ready reference the data set is given below:

0.1, 0.33, 0.44, 0.56, 0.59, 0.72, 0.74, 0.77, 0.92, 0.93, 0.96, 1, 1, 1.02, 1.05, 1.07, 1.07, 1.08, 1.08, 1.08, 1.09, 1.12, 1.13, 1.15, 1.16, 1.2, 1.21, 1.22, 1.22, 1.24, 1.3, 1.34, 1.36, 1.39, 1.44, 1.46, 1.53, 1.59, 1.6, 1.63, 1.63, 1.68, 1.71, 1.72, 1.76, 1.83, 1.95, 1.96, 1.97, 2.02, 2.13, 2.15,

2.16, 2.22, 2.3, 2.31, 2.4, 2.45, 2.51, 2.53, 2.54, 2.54, 2.78, 2.93, 3.27, 3.42, 3.47, 3.61, 4.02, 4.32, 4.58, 5.55.

Before illustrating the methodology, our model is tested for goodness of fit. The maximum likelihood estimates of the three parameters of OGELLD for the survival times of guinea pigs data are $\hat{\lambda}=1.1513, \hat{\theta}=1.1606$ and $\hat{\gamma}=2.6538$. Using the Kolmogorov-Smirnov test, we found that the maximum distance between the data and the fitted OGELLD is 0.089 with p-value 0.617. Thus, the three parameter OGELLD provides a reasonable fit for the survival times of guinea pigs data. The goodness of fit for our model is emphasized by plotting the density plot and Q-Q plot displayed in Figure 1. The plan parameters are also computed at fitted parametric values and are displayed in Tables 5 for 50th percentiles. Suppose that it is desired to develop a group acceptance sampling plan to satisfy that the 50th percentile lifetime is greater than survival times of guinea pigs 0.20 days through the experiment to be completed survival times of guinea pigs by 0.40 days to protect the producer's risk at 5%. For $\hat{\lambda}=1.1513, \hat{\theta}=1.1606$ and $\hat{\gamma}=2.6538$, the consumer's risk is $\beta=0.25$, r=5, $\delta_q^0=0.5$ and t_q/t_q^0=2, the minimum number of groups and the acceptance number are g=6 and c=2 from Table 5. Thus, the design can be implemented as follows: select a total of 30 guinea pigs and allocate five guinea pigs to each of the 6 groups. We can accept the lot when no more than two failures occur before survival times of guinea pigs 0.40 days from each of the 6 groups. According to this plan, the survival times of guinea pigs could have been accepted because there are only two failures before the termination time 0.40 days.

5. Comparison of distributions

In Table 7, we compare the plan parameters of the proposed group acceptance sampling plan with the generalized log-logistic distribution (GLLD) studied by Aslam *et al*. (2011b) and odds exponential log-logistic distribution (OELLD) studied by Rosaiah *et al.* (2016a), when $\beta=0.10$ and $r=5, \delta_q^0=0.5$. The acceptance number for the OGELLD is smaller as compared to GLLD and OELLD for 50th percentile.

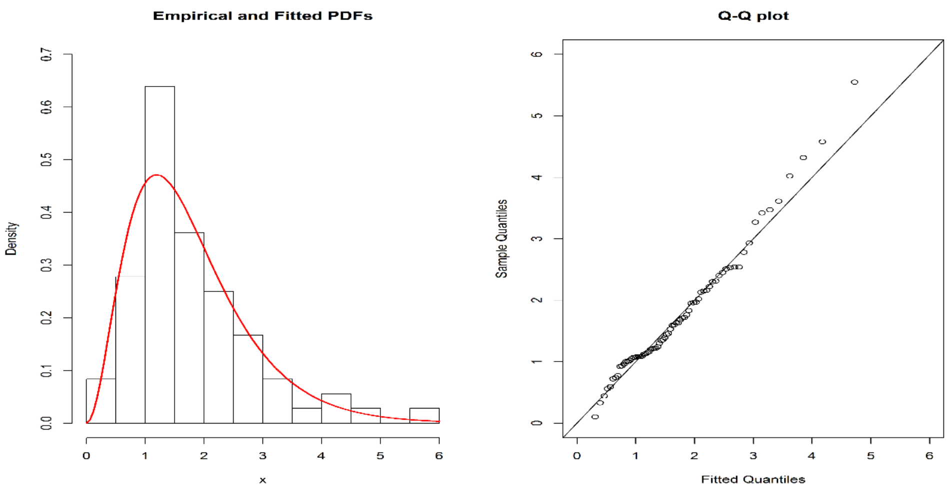

Figure 1. The density plot and Q-Q plot of the fitted OGELLD for the survival times of guinea pigs data.

6. Conclusions

In this article, a group acceptance sampling plan is developed when the lifetime of the product follows OGELLD. The plan parametric quantities like the number of groups, ‘*g*’, and the acceptance number ‘*c*’ are determined by considering the consumer's risk and producer's risk simultaneously. Our proposed plan noticed that if the percentile ratio increases, the number of groups ‘*g*’ reduces and as ‘*r*’ increases the number of groups reduces for all the parametric combinations considered in this article. The proposed plan is illustrated with a real lifetime data set in health sciences, survival times of guinea pigs in days, and results show that our methodology performs well as compared with existing sampling plans.

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APENNDIX

Table 1. GASP for OGELLD with $\lambda = 2, \theta = 2$ and $\gamma = 2$ for 50th percentile

β	t_q / t_q^0	r=5						r=10					
		δ_q =0.50			δ_q =1.0			δ_q =0.50			δ_q =1.0		
		c	g	$P_a(p)$	c	g	$P_a(p)$	c	g	$P_a(p)$	c	g	$P_a(p)$
0.25	2	1	8	0.9797	3	4	0.9531	1	4	0.9797	-	-	-
	4	0	4	0.9928	0	1	0.9730	0	2	0.9928	1	2	0.9947
	6	0	4	0.9986	0	1	0.9944	0	2	0.9986	0	1	0.9888
	8	0	4	0.9995	0	1	0.9982	0	2	0.9995	0	1	0.9964
0.10	2	1	11	0.9635	3	4	0.9531	1	6	0.9572	-	-	-
	4	0	7	0.9874	0	1	0.9730	0	4	0.9857	1	2	0.9947
	6	0	7	0.9975	0	1	0.9944	0	4	0.9971	0	1	0.9888
	8	0	7	0.9992	0	1	0.9982	0	4	0.9991	0	1	0.9964
0.05	2	2	18	0.9866	3	4	0.9531	2	9	0.9866	-	-	-
	4	0	9	0.9839	0	1	0.9730	0	5	0.9821	1	2	0.9947
	6	0	9	0.9968	0	1	0.9944	0	5	0.9964	0	1	0.9888
	8	0	9	0.9990	0	1	0.9982	0	5	0.9989	0	1	0.9964
0.01	2	2	24	0.9715	3	4	0.9531	2	12	0.9715	-	-	-
	4	0	13	0.9768	1	3	0.9970	0	7	0.9750	1	2	0.9947
	6	0	13	0.9953	0	2	0.9888	0	7	0.9950	0	1	0.9888
	8	0	13	0.9985	0	2	0.9964	0	7	0.9984	0	1	0.9964

Table 2. GASP for OGELLD with $\lambda = 2, \theta = 1.5$ and $\gamma = 1.5$ for 50th percentile

β	t_q / t_q^0	r=5						r=10					
		δ_q =0.50			δ_q =1.0			δ_q =0.50			δ_q =1.0		
		c	g	$P_a(p)$	c	g	$P_a(p)$	c	g	$P_a(p)$	c	g	$P_a(p)$
0.25	2	3	7	0.9502	-	-	-	4	5	0.9513	-	-	-
	4	1	4	0.9864	2	3	0.9798	1	2	0.9864	4	5	0.9513
	6	0	2	0.9643	1	2	0.9885	1	2	0.9976	1	2	0.9564
	8	0	2	0.9810	1	2	0.9966	1	2	0.9993	1	2	0.9864
0.10	2	4	10	0.9513	-	-	-	4	5	0.9513	-	-	-
	4	1	5	0.9792	2	3	0.9798	1	3	0.9707	4	5	0.9513
	6	1	5	0.9963	1	2	0.9885	1	3	0.9946	1	2	0.9564
	8	1	5	0.9989	1	2	0.9966	1	3	0.9985	1	2	0.9864
0.05	2	5	13	0.9548	-	-	-	6	8	0.9591	-	-	-
	4	1	6	0.9707	2	3	0.9798	1	3	0.9707	4	5	0.9513
	6	1	6	0.9946	1	2	0.9885	1	3	0.9946	1	2	0.9564
	8	1	6	0.9985	1	2	0.9966	1	3	0.9985	1	2	0.9864
0.01	2	7	19	0.9634	-	-	-	7	10	0.9528	-	-	-
	4	1	8	0.9504	2	3	0.9798	1	4	0.9504	4	5	0.9513
	6	1	8	0.9906	1	3	0.9746	1	4	0.9906	1	2	0.9564
	8	1	8	0.9973	1	3	0.9923	1	4	0.9973	1	2	0.9864

Table 3. GASP for OGELLD with $\lambda = 2, \theta = 2$ and $\gamma = 2$ for 25th percentile

β	t_q / t_q^0	*r=5*						*r=10*					
		δ_q =0.50			δ_q =1.0			δ_q =0.50			δ_q =1.0		
		c	*g*	$P_a(p)$	*c*	*g*	$P_a(p)$	*c*	*g*	$P_a(p)$	*c*	*g*	$P_a(p)$
0.25	2	1	22	0.9830	1	2	0.9748	1	11	0.9830	2	3	0.9604
	4	0	11	0.9936	0	1	0.9910	0	6	0.9931	0	1	0.9822
	6	0	11	0.9987	0	1	0.9980	0	6	0.9986	0	1	0.9964
	8	0	11	0.9996	0	1	0.9994	0	6	0.9996	0	1	0.9988
0.10	2	1	31	0.9678	2	4	0.9866	1	16	0.9659	2	3	0.9604
	4	0	18	0.9896	0	2	0.9822	0	9	0.9896	0	1	0.9822
	6	0	18	0.9979	0	2	0.9964	0	9	0.9979	0	1	0.9964
	8	0	18	0.9993	0	2	0.9988	0	9	0.9993	0	1	0.9988
0.05	2	1	38	0.9535	2	5	0.9754	1	19	0.9535	2	3	0.9604
	4	0	24	0.9862	0	3	0.9734	1	12	0.9862	0	2	0.9647
	6	0	24	0.9972	0	3	0.9946	0	12	0.9972	0	2	0.9928
	8	0	24	0.9991	0	3	0.9983	0	12	0.9991	0	2	0.9977
0.01	2	2	66	0.9777	3	8	0.9818	2	33	0.9777	3	4	0.9818
	4	0	36	0.9793	0	4	0.9647	0	18	0.9793	0	2	0.9647
	6	0	36	0.9959	0	4	0.9928	0	18	0.9959	0	2	0.9928
	8	0	36	0.9987	0	4	0.9977	0	18	0.9987	0	2	0.9977

Table 4. GASP for OGELLD with $\lambda = 2, \theta = 1.5$ and $\gamma = 1.5$ for 25th percentile

β	t_q / t_q^0	r=5						r=10					
		δ_q =0.50			δ_q =1.0			δ_q =0.50			δ_q =1.0		
		c	g	$P_a(p)$	c	g	$P_a(p)$	c	g	$P_a(p)$	c	g	$P_a(p)$
0.25	2	3	16	0.9663	3	04	0.9603	3	8	0.9663	-	-	-
	4	1	8	0.9923	1	02	0.9905	1	4	0.9923	1	2	0.9636
	6	0	5	0.9673	1	02	0.9983	1	4	0.9987	1	2	0.9932
	8	0	5	0.9827	1	02	0.9995	1	4	0.9996	1	2	0.9980
0.10	2	4	24	0.9635	4	06	0.9547	4	12	0.9635	-	-	-
	4	1	12	0.9832	1	03	0.9788	1	6	0.9832	1	2	0.9636
	6	0	7	0.9545	1	03	0.9962	1	6	0.9970	1	2	0.9936
	8	0	7	0.9757	1	03	0.9989	1	6	0.9992	1	2	0.9936
0.05	2	5	32	0.9640	5	08	0.9534	5	16	0.9640	-	-	-
	4	1	14	0.9775	1	04	0.9636	1	7	0.9775	1	2	0.9636
	6	1	14	0.9960	1	04	0.9932	1	7	0.9960	1	2	0.9932
	8	1	14	0.9989	1	04	0.9980	1	7	0.9989	1	2	0.9980
0.01	2	7	48	0.9687	7	12	0.9564	7	24	0.9687	-	-	-
	4	1	20	0.9568	2	07	0.9841	1	10	0.9568	4	5	0.9773
	6	1	20	0.9920	1	05	0.9895	1	10	0.9920	1	2	0.9850
	8	1	20	0.9977	1	05	0.9969	1	10	0.9977	1	2	0.9950

Table 5. GASP for OGELLD with $\hat{\lambda}=1.1513, \hat{\theta}=1.1606$ and $\hat{\gamma}=2.6538$ for 50th percentile

β	t_q/t_q^0	*r=5*						*r=10*					
		δ_q=0.50			δ_q=1.0			δ_q=0.50			δ_q=1.0		
		c	*g*	$P_a(p)$	*c*	*g*	$P_a(p)$	*c*	*g*	$P_a(p)$	*c*	*g*	$P_a(p)$
0.25	2	2	6	0.9553	-	-	-	2	3	0.9553	-	-	-
	4	0	2	0.9620	1	2	0.9724	0	1	0.9620	2	3	0.9553
	6	0	2	0.9882	0	1	0.9569	0	1	0.9882	1	2	0.9868
	8	0	2	0.9950	0	1	0.9808	0	1	0.9950	0	1	0.9620
0.10	2	3	9	0.9687	-	-	-	3	5	0.9563	-	-	-
	4	1	6	0.9939	1	2	0.9724	1	3	0.9939	2	3	0.9553
	6	0	3	0.9824	0	1	0.9569	0	2	0.9766	1	2	0.9868
	8	0	3	0.9925	0	1	0.9808	0	2	0.9900	0	1	0.9620
0.05	2	4	13	0.9706	-	-	-	4	7	0.9613	-	-	-
	4	1	7	0.9918	1	2	0.9724	1	4	0.9894	2	3	0.9553
	6	0	4	0.9766	0	1	0.9569	0	2	0.9766	1	2	0.9868
	8	0	4	0.9900	0	1	0.9808	0	2	0.9900	0	1	0.9620
0.01	2	5	18	0.9669	-	-	-	5	9	0.9669	-	-	-
	4	1	9	0.9867	2	3	0.9933	1	5	0.9838	2	3	0.9553
	6	0	11	0.9823	0	2	0.9713	0	6	0.9808	0	1	0.9713
	8	0	11	0.9929	0	2	0.9884	0	6	0.9922	0	1	0.9884

Table 6. Comparison between GLLD, OELLD and OGELLD

t_q/t_q^0	GLLD			OELLD			OGELLD		
	c	g	$P_a(p_1)$	c	g	$P_a(p_1)$	c	g	$P_a(p_1)$
2	1	69	0.9959	5	12	0.9587	4	10	0.9513
4	0	41	0.9990	1	5	0.9705	1	5	0.9792
6	0	41	0.9999	1	5	0.9936	1	5	0.9963
8	0	41	0.9999	0	3	0.9602	1	5	0.9989